

Mechanism of temperature dependence of RIXS spectra in charge transfer insulators

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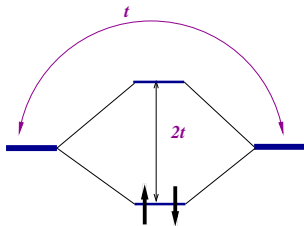
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- T - dependence of charge response of strongly correlated systems
- O-K RIXS calculation
- Analytic consideration of "Cu-O-Cu" cluster
- RIXS spectra for ESC
- Conclusions



H₂ molecule, Hückel MO approach



$$|\psi_{\pm}\rangle = \frac{1}{\sqrt{2}}[\phi_1(r) \pm \phi_2(r)],$$

$$|g\rangle = a_{+, \uparrow}^{\dagger} a_{-, \downarrow}^{\dagger} |vac\rangle$$

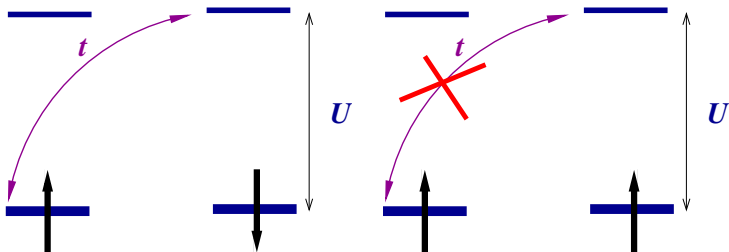
$$\Delta E = E_1 - E_0 = 2t$$

$$P_2 = P_{11}$$

Changes in charge response are expected at $T \sim 2t$



H₂ molecule, H-L approach



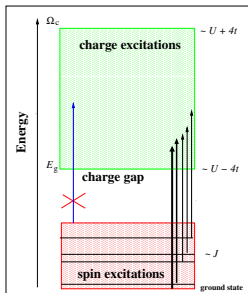
$$|g\rangle = \frac{1}{\sqrt{2}} \left[a_{1,\uparrow}^\dagger a_{2,\downarrow}^\dagger - a_{1,\downarrow}^\dagger a_{2,\uparrow}^\dagger \right] |vac\rangle - \frac{2t}{U} |si\rangle, \quad P_2 < P_{11}$$

$$|t\rangle = a_{1,\uparrow}^\dagger a_{2,\uparrow}^\dagger |vac\rangle, \quad \Delta E = E_1 - E_0 = \frac{4t^2}{U} = J \ll t$$

Changes in charge response are expected at $T \sim J$



Mott insulator



$$\hat{H} = \hat{K} + \hat{U},$$

$$\hat{K} = -t \sum_{\langle i,j \rangle, \sigma} d_{i,\sigma}^\dagger d_{j,\sigma},$$

$$\hat{U} = U \sum_i d_{i,\uparrow}^\dagger d_{i,\uparrow} d_{i,\downarrow}^\dagger d_{i,\downarrow}$$

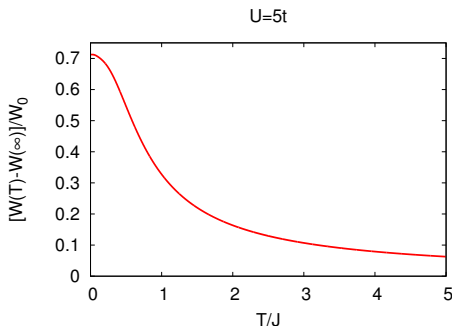
$$\sigma_{\alpha\alpha}(\omega) = \frac{1}{V} \langle\langle \hat{P}^\alpha | \hat{J}^\alpha \rangle\rangle$$

$$W(T) = \frac{\tilde{\omega}_p^2}{8} = \frac{\pi}{2V} \langle\langle [\hat{P}^\alpha, \hat{H}], \hat{P}^\alpha \rangle\rangle = -\frac{\pi e^2 a \langle\hat{K}\rangle}{2Nd^2 \hbar^2}$$



1D Hubbard model

$$W(T) = -\frac{\pi e^2 a}{2d^2 \hbar^2} (t \partial_t J) \left(C_\delta(T) - \frac{1}{4} \right),$$

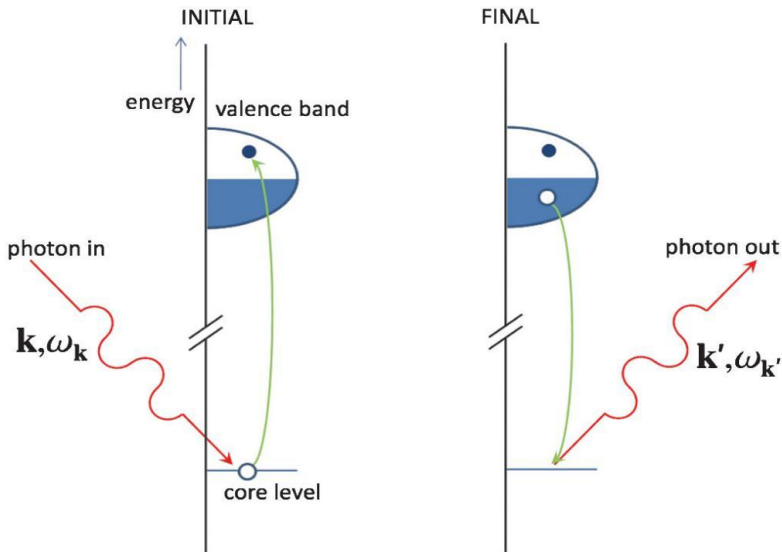


$$t \partial_t J = \frac{8t^2}{U} - \dots$$



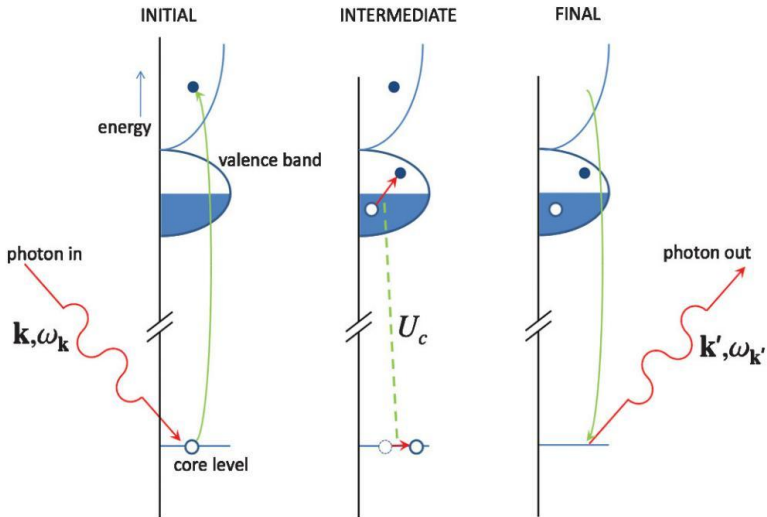
Resonant Inelastic X-ray Scattering (RIXS)

Direct process



Resonant Inelastic X-ray Scattering (RIXS)

Indirect process



Scattering cross section

wrt the solid angle $\Omega_{\mathbf{k}_2}$ and energy ω

$$\begin{aligned}\frac{d^2\sigma}{d\Omega_{\mathbf{k}_2}d\omega} &= \frac{\omega^2}{c^4} \left(\frac{1}{2\pi}\right)^3 W_{12}, \\ W_{12} &= \sum_f \frac{(2\pi)^3}{\Omega\omega} \left(\frac{e^2}{m}\right)^2 \delta(E_g + \Omega - E_f - \omega) \\ &\times \left| \langle f | \rho_{\mathbf{k}_1 - \mathbf{k}_2} | g \rangle \sum_{\mu} \epsilon'_{\mu} \epsilon_{\mu} \right. \\ &+ \frac{1}{m} \sum_{i, \mu, \nu} \left(\frac{\langle f | \epsilon'_{\mu} p_{\mu}(\mathbf{k}_2) | i \rangle \langle i | \epsilon_{\nu} p_{\nu}(-\mathbf{k}_1) | g \rangle}{E_i - \Omega - E_g} \right. \\ &\left. \left. + \frac{\langle f | \epsilon_{\nu} p_{\nu}(\mathbf{k}_1) | i \rangle \langle i | \epsilon'_{\mu} p_{\mu}(-\mathbf{k}_2) | g \rangle}{E_i + \omega - E_g} \right) \right|^2,\end{aligned}$$

where $\mathbf{p}(\mathbf{k}) = \sum_n \mathbf{p}_n \exp(-i\mathbf{k} \cdot \mathbf{r}_n)$, $\rho(\mathbf{k}) = \sum_n \exp(-i\mathbf{k} \cdot \mathbf{r}_n)$



$$\begin{aligned}
 I(\Omega, \omega, \vec{\epsilon}, \vec{\epsilon}') &\propto \sum_f |F_{fg}(\Omega, \vec{\epsilon}, \vec{\epsilon}')|^2 \delta[\Omega - \omega - (E_f - E_g)] \\
 F_{fg}(\Omega, \vec{\epsilon}, \vec{\epsilon}') &= \sum_i \frac{\langle f | \epsilon'_\mu p_\mu(\mathbf{k}_2) | i \rangle \langle i | \epsilon_\nu p_\nu(-\mathbf{k}_1) | g \rangle}{E_g + \Omega - E_i - i\Gamma} \\
 &= \langle f | \hat{D}^\dagger \hat{G}(E_g + \Omega - i\Gamma) \hat{D} | g \rangle, \\
 \hat{G}(z) &= \sum_i \frac{|i\rangle \langle i|}{z - E_i} = (z - \hat{H})^{-1}
 \end{aligned}$$

$\Gamma \sim 1$ eV is the spectral broadening due to the core- hole lifetime



O K -edge RIXS spectrum

$$F_{fg}(\Omega, \vec{\epsilon}, \vec{\epsilon}') = \sum_{\mathbf{R}, m, \mu, \nu} \frac{\langle f | \epsilon'_\mu \hat{T}_{\mu, \mathbf{R}} | m, \mathbf{R} \rangle \langle m, \mathbf{R} | \epsilon_\nu \hat{T}_{\nu, \mathbf{R}} | g \rangle}{E_g + \Omega - E_{m, \mathbf{R}} - i\Gamma}$$
$$\hat{T}_{\mu, \mathbf{R}} = \sum_{\sigma} s_{\mathbf{R}\sigma}^\dagger p_{\mathbf{R}\mu\sigma} + h.c.$$
$$\hat{H}_{\mathbf{R}} = \hat{H}_{pd} + \hat{H}_{C, \mathbf{R}},$$
$$\hat{H}_{C, \mathbf{R}} = \varepsilon_s \sum_{\sigma} s_{\mathbf{R}\sigma}^\dagger s_{\mathbf{R}\sigma} + U_C \sum_{\sigma, \sigma', \mu} s_{\mathbf{R}\sigma}^\dagger s_{\mathbf{R}\sigma} p_{\mathbf{R}\mu\sigma'}^\dagger p_{\mathbf{R}\mu\sigma'}.$$



"Alchemical" approximation

$$\hat{H}'_{pd,\mathbf{R}} = \hat{H}_{pd} + U_C \sum_{\sigma,\alpha} p_{\mathbf{R}\alpha\sigma}^\dagger p_{\mathbf{R}\alpha\sigma}$$

$$U_C \sim \epsilon_{O,2p} - \epsilon_{F,2p} \sim 3 - 5 \text{ eV},$$

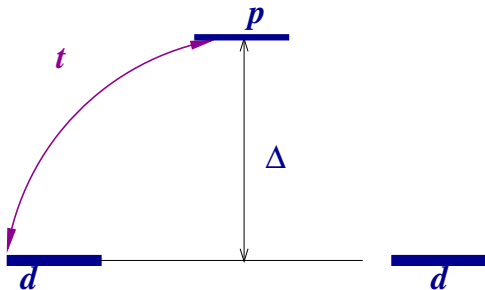
$$I_g(\Omega, \omega, \vec{\epsilon}, \vec{\epsilon}') = \sum_f \left| \sum_{\mathbf{R}, \mu, \nu} \epsilon'_\mu \epsilon_\nu M_{\mathbf{R}\mu\nu}^{fg}(\Omega) \right|^2 \delta(E_g + \Omega - E_f - \omega),$$

$$M_{\mathbf{R}\mu\nu}^{fg}(\Omega) = \sum_\sigma \langle f | p_{\mathbf{R}\mu\sigma}^\dagger \frac{1}{z - \hat{H}'_{pd,\mathbf{R}} - \epsilon_s} p_{\mathbf{R}\nu\sigma} | g \rangle,$$

$$z \equiv E_g + \Omega - i\Gamma.$$



"Cu-O-Cu" cluster



$$\hat{H}_{pd} = \Delta \sum_{\sigma} p_{\sigma}^{\dagger} p_{\sigma} + t \sum_{i,\sigma} \left(\tilde{d}_{i,\sigma}^{\dagger} p_{\sigma} + p_{\sigma}^{\dagger} \tilde{d}_{i,\sigma} \right)$$

$$\tilde{d}_{i,\sigma} \equiv d_{\sigma} \left(1 - d_{-\sigma}^{\dagger} d_{-\sigma} \right)$$



Singlet sector, $S = 0$

$$|s_d\rangle = \frac{1}{\sqrt{2}} \left(\tilde{d}_{1,\uparrow}^\dagger \tilde{d}_{2,\downarrow}^\dagger - \tilde{d}_{1,\downarrow}^\dagger \tilde{d}_{2,\uparrow}^\dagger \right) |vac\rangle + \dots, \quad \phi = \arcsin \frac{R}{\sqrt{Q^3}},$$

$$\begin{array}{c} \uparrow \\ \downarrow \end{array} \quad \begin{array}{c} \uparrow \\ \downarrow \end{array} \quad E_0 = \Delta - 2\sqrt{Q} \left(\frac{\sqrt{3}}{2} \cos \frac{\varphi}{3} + \frac{1}{2} \sin \frac{\varphi}{3} \right) \approx -\frac{2t^2}{\Delta},$$

$$|ZRS_s\rangle = \frac{1}{\sqrt{2}} [|ZRS, 1\rangle + |ZRS, 2\rangle] + \dots, \quad R = -t^2 \Delta$$

$$\begin{array}{c} \uparrow \\ \downarrow \end{array} \quad |ZRS, i\rangle = \frac{1}{\sqrt{2}} \left(\tilde{d}_{i,\uparrow}^\dagger p_{i,\downarrow}^\dagger - \tilde{d}_{i,\downarrow}^\dagger p_{i,\uparrow}^\dagger \right) |vac\rangle$$

$$\begin{array}{c} \uparrow \\ \downarrow \end{array} \quad \begin{array}{c} \uparrow \\ \downarrow \end{array} \quad E_1 = \Delta + 2\sqrt{Q} \sin \frac{\varphi}{3} \approx \Delta - \frac{2t^2}{\Delta} \left(1 - \frac{6t^2}{\Delta^2} \right),$$

$$|s_p\rangle = p_{\uparrow}^\dagger p_{\downarrow}^\dagger |vac\rangle + \dots, \quad Q = \frac{1}{3} (\Delta^2 + 6t^2),$$

$$\begin{array}{c} \uparrow \\ \uparrow \\ \downarrow \\ \downarrow \end{array} \quad E_2 = \Delta + 2\sqrt{Q} \left(\frac{\sqrt{3}}{2} \cos \frac{\varphi}{3} - \frac{1}{2} \sin \frac{\varphi}{3} \right)$$

$$\begin{array}{c} \uparrow \\ \downarrow \end{array} \quad \begin{array}{c} \uparrow \\ \downarrow \end{array} \quad \approx 2\Delta + \frac{4t^2}{\Delta} - \frac{12t^4}{\Delta^3}.$$



Triplet sector, $S = 1, S^z = 1$

$$|t_d\rangle = \tilde{d}_{1,\uparrow}^\dagger \tilde{d}_{2,\uparrow}^\dagger |vac\rangle + \dots,$$

$$\begin{array}{c} \uparrow \\ d \end{array} \quad \begin{array}{c} \uparrow \\ d \end{array} \quad E_{t,0} = \frac{\Delta}{2} (1 - R_t) \approx -\frac{2t^2}{\Delta} + \frac{4t^4}{\Delta^3},$$

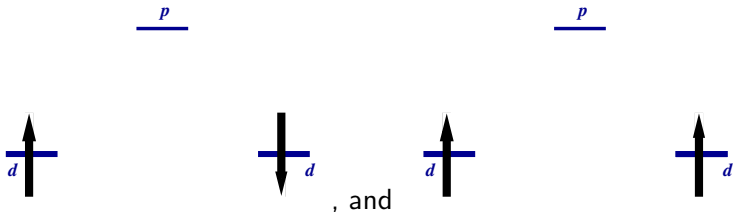
$$|ZRT\rangle = \frac{1}{\sqrt{2}} p_\uparrow^\dagger (\tilde{d}_{2,\uparrow}^\dagger - \tilde{d}_{1,\uparrow}^\dagger) |vac\rangle + \dots,$$

$$\begin{array}{c} \uparrow \\ d \end{array} \quad \begin{array}{c} \uparrow \\ p \end{array} \quad \begin{array}{c} \uparrow \\ d \end{array} \quad E_{t,1} = \frac{\Delta}{2} (1 + R_t) \approx \Delta + \frac{2t^2}{\Delta} - \frac{4t^4}{\Delta^3},$$



Low-energy spectrum. "Magnetic" states

In the subspace of the states $|s_d\rangle$ and $|t_d\rangle \approx$,



the system has only spin degrees of freedom, which are described by an effective Heisenberg Hamiltonian

$$\begin{aligned}\hat{J} &= J\hat{\mathbf{S}}_1 \cdot \hat{\mathbf{S}}_2 + 2e_d, \\ J &= E_{t,0} - E_0 \approx 4t^4/\Delta^3 \ll t, \Delta, \\ 2e_d &= -2t^2/\Delta.\end{aligned}$$



The eigenenergies of even states are

$$E_{\pm} = \frac{\Delta + U_C}{2} \pm \sqrt{1 + 8 [t / (\Delta + U_C)]^2},$$

An odd state

$$|\sigma, a\rangle = \frac{1}{\sqrt{2}} \left(\tilde{d}_{1,\sigma 0}^{\dagger} - \tilde{d}_{2,\sigma 0}^{\dagger} \right) |vac\rangle$$

has the energy

$$E_a = 0.$$



O K RIXS spectrum for finite temperature

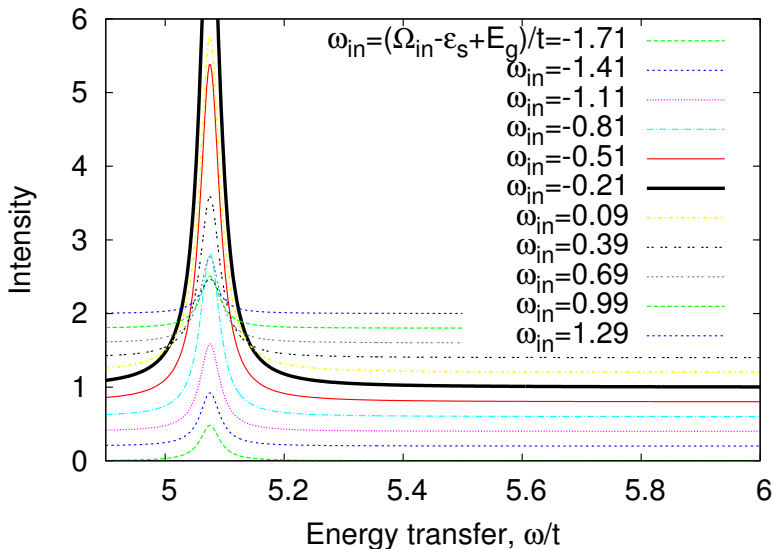
$$I(T, \Omega, \omega, \vec{\epsilon}, \vec{\epsilon}') = \langle I_g(\Omega, \omega, \vec{\epsilon}, \vec{\epsilon}') \rangle_T \\ = \frac{\sum_g \exp(-E_g/kT) I_g(\Omega, \omega, \vec{\epsilon}, \vec{\epsilon}')}{\sum_g \exp(-E_g/kT)},$$

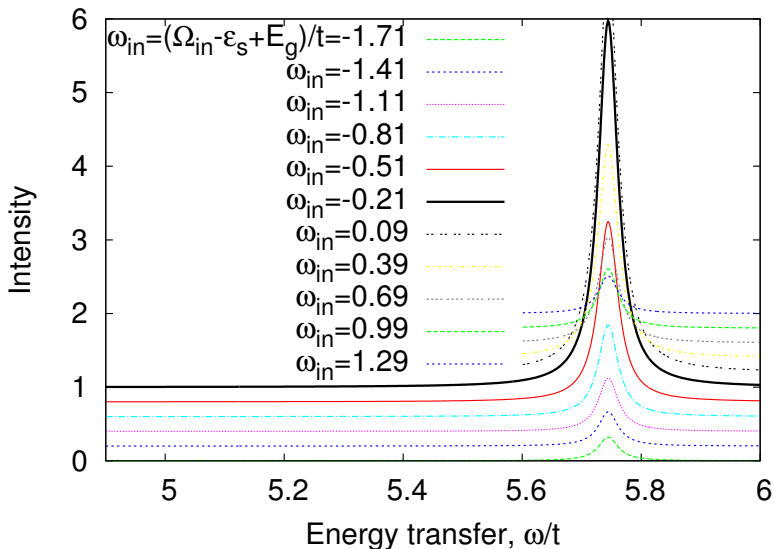
For $T \sim J \ll \Delta$

$$I(T, \Omega, \omega) = w_s I_0(\Omega, \omega) + w_t I_t(\Omega, \omega),$$

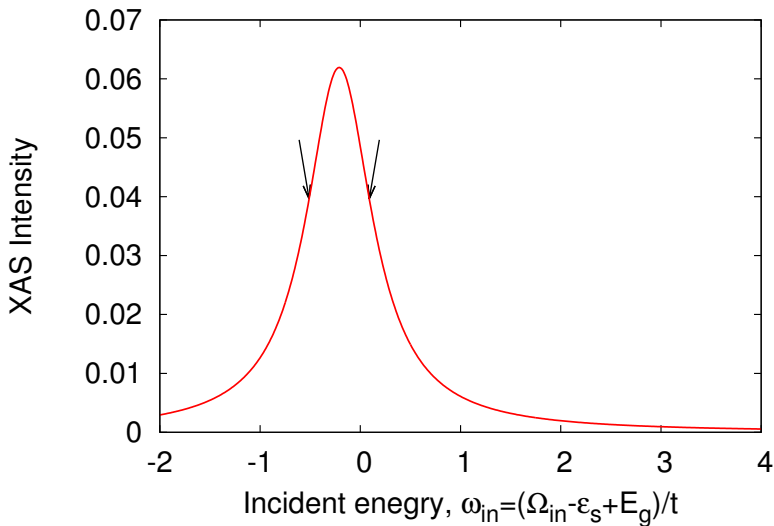
where $w_s = 1/Q(T)$, $w_t = 3 \exp(-J/kT)/Q(T)$,
 $Q(T) = 1 + 3 \exp(-J/kT)$.



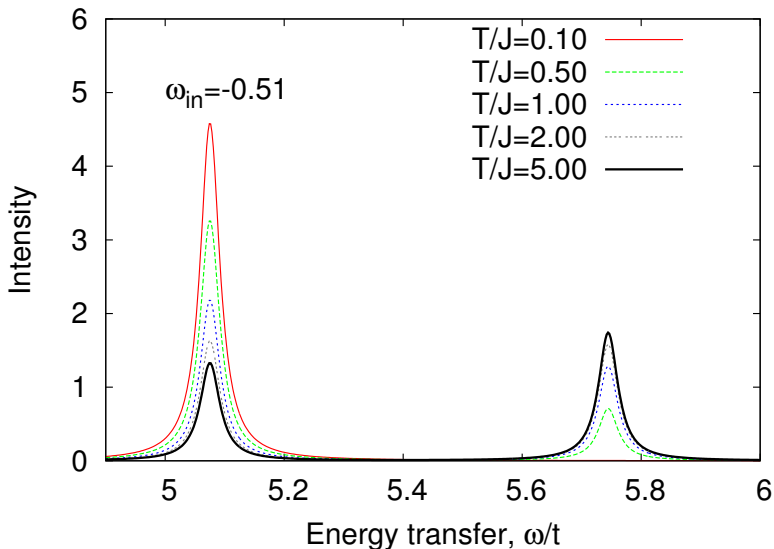




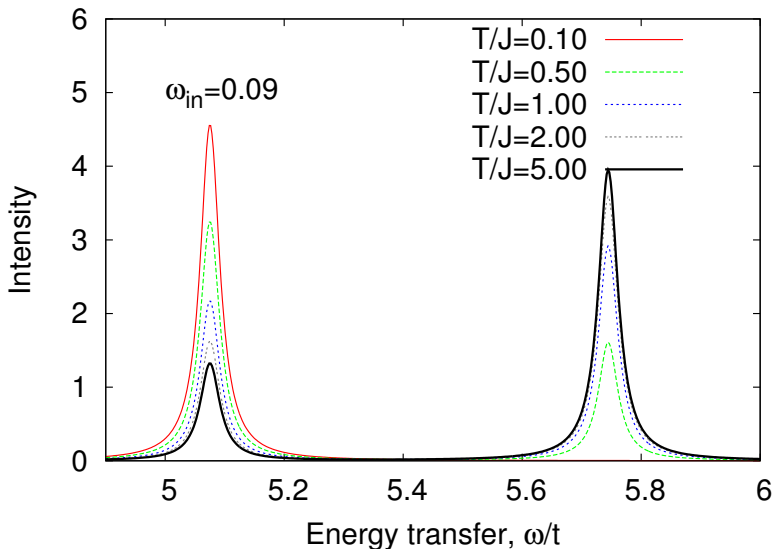
XAS spectrum



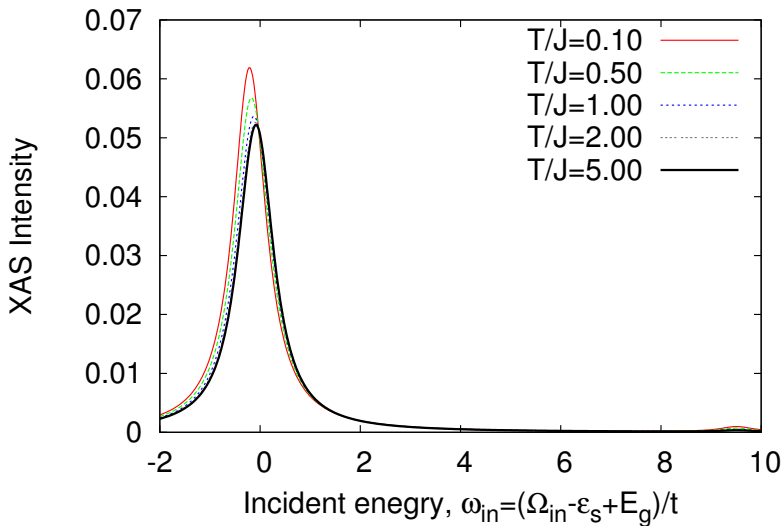
T-dependence of RIXS spectra



T-dependence of RIXS spectra

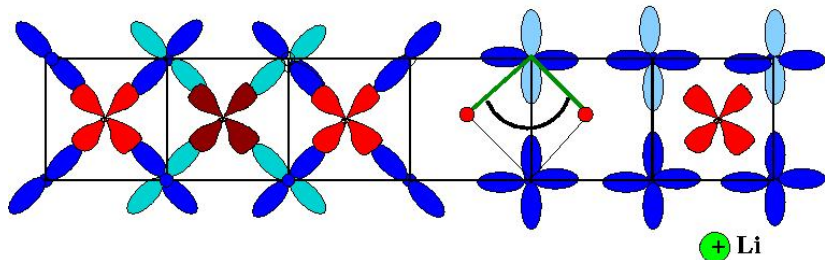


T-dependence of XAS spectrum



Orbitals and exchange in edge-shared CuO_2 chains

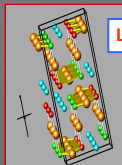
$4+, 3+$ Ge, Si, La



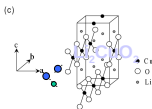
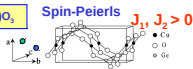
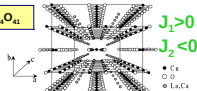
Edge-shared CuO_2 chains: large variety of ground states

Different embeddings (Interchain coupling, crystal field) Cu-O-Cu bond angles, spin anisotropies

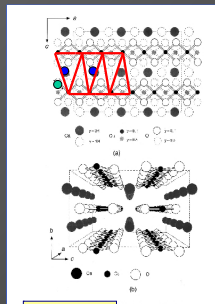
in-chain-spiral



off-chain spiral



AFM/FM
 $J_1 < 0, J_2 > 0$



1. **fm in-chain & afm off-chain ordering:** Li_2CuO_2 , $\text{Ca}_8\text{La}_6\text{Cu}_{24}\text{O}_{41}$, $\text{Ca}_2\text{Y}_2\text{Cu}_5\text{O}_{10}$
2. **No spiral ordering, but frustrated:** $\text{Li}_2\text{ZrCuO}_4$, $\text{Rb(Cs)}_2\text{Cu}_2\text{Mo}_3\text{O}_{12}$
3. **Incommensurable (IC) in-chain ordering:** in-chain helix: $\text{Li(Na)Cu}_2\text{O}_2$, LiVCuO_4 , CuSiO_3
4. **Incommensurable off-chain**

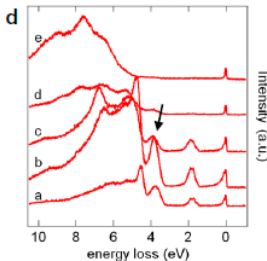
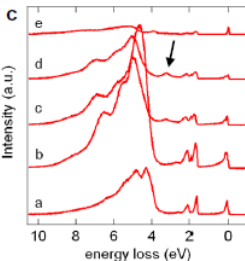
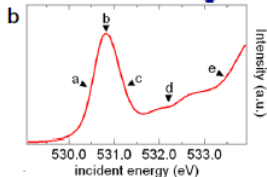
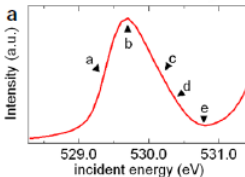
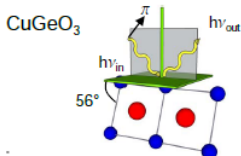
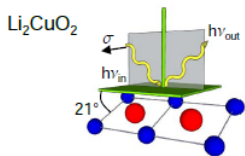
1D-spiral:

$$\alpha = |J_2/J_1| \geq 1/4, \cos \varphi = -J_1/4J_2$$

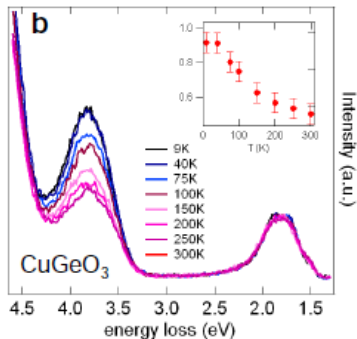
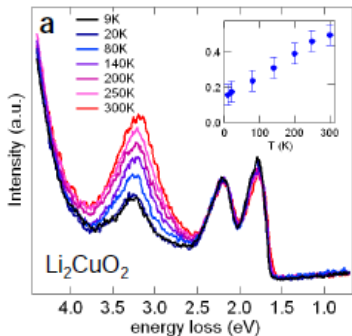
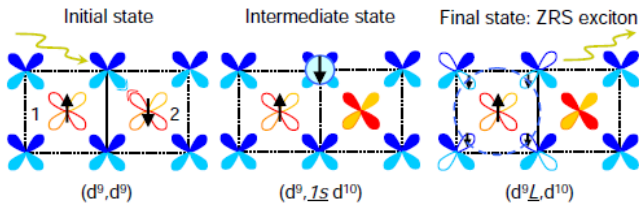
$$\text{if } J_2/|J_1| \gg 1, \varphi \approx \pi/2$$

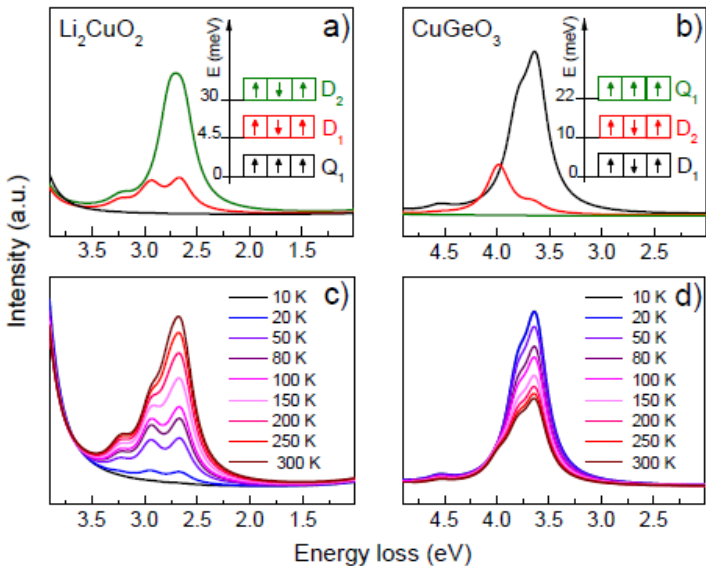
3D: FM maybe stabilized by specific AFM interchain coupling and easy-axis spin anisotropy





T-dependence of ZRS peak





- In strongly correlated systems, the probability of charge-transfer excitations depends on the magnetic state of the system.
- RIXS spectra of antiferromagnetic CTI strongly change in the temperature interval between zero and several J



Thank you for your attention!

